Seven-branes and instantons in type IIB supergravity

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Introduction

The subject of this thesis will be introduced. The studies performed in the coming chapters attempt to improve on the understanding of certain aspects of string theory. Here, I will give a personal point of view on the subject of string theory after which I will briefly sketch the most elementary properties of string theory that allow me to further introduce the subject of this thesis.

The aim of theoretical physics is to describe the observable world around us through mathematically consistent theories in which physical quantities are derived from first principles. By the observable world is meant the collection of things whose existence has been experimentally established. The 'first principles' are postulated by the scientific community and reflect properties of the mathematical theories that are not disproved by experiment.

The above attempt to describe the arena of theoretical physics is necessarily imprecise, because research in theoretical physics sometimes forces one to change one's perspective on its very nature. For example the advent of quantum theory strongly influenced the way we think about observable quantities. The advent of general relativity and input from cosmological observations influenced our notion of the 'world around us'. Undoubtedly, this will go on as science progresses.

An important aspect of the above attempted description of theoretical physics is the need for a mathematically consistent theory. There are generally speaking two approaches to the construction of new theories. One method goes by modeling large amounts of experimental data obtained from measurements at a particular energy scale. Once a consistent model is found that reproduces the data to within the observed accuracy, one can extrapolate the model to higher energy scales to see what predictions it gives. This may be referred to as a bottom-up approach. This approach has the advantage of being strongly correlated to experiment and the disadvantage that it is by construction energy scale dependent making it difficult to construct a model that is valid at all energy scales. The other approach starts by postulating aspects that we (a particular scientific community in which consensus has been reached) wish to attribute to nature, because they have found to be rigorously true in experiments (possibly performed at different energy scales) and to construct a mathematically consistent theory around these postulates that is valid at all energy scales. This approach may be referred to as a top-down approach. It has the advantage of being applicable at all energy scales and the disadvantage that it does not necessarily (by construction) lead to a connection with experiment. A connection with experiment will only be possible if the theory makes a unique prediction about what happens at an experimentally accessible energy scale.

Two beautiful examples of a bottom-up approach are the standard model of elementary particles constructed by Glashow, Salam and Weinberg and the so-called ΛCDM model of cosmology that is used to describe the currently observed cosmological data such as the dark energy and dark matter components of our universe. In the abbreviation ΛCDM the Λ denotes the cosmological constant parameterizing the dark energy and CDM stands for cold dark matter.

A particularly nice example of a top-down approach (within the context of classical physics) is Einstein's theory of general relativity. In the context of quantum theory an example is provided by quantum field theory which derives from a small number of postulates and can sometimes be shown to apply at all energy scales[1]. The top-down approach is most useful in finding the mathematical arena in which a physical theory can be formulated.

Quantum field theory is not a unique theory, because it does not specify its own field content and neither does it specify the properties of the space-time on which the fields are propagating. Another drawback of quantum field theory is that it has proven, at least up to date, only possible to describe gravity at some energy scale as an effective field theory. This situation is markedly different in string theory. String theory is another top-down approach that is at present still under construction in that its complete quantum mechanical formulation is not fully known. However, there is very strong evidence that the generalization of string theory, called M-theory, is a unique theory that is valid at all energy scales, contains gravity and predicts the dimensionality of the world in which extended objects, generally called branes, are moving to be 11-dimensional space-time. Further, it has been shown that many quantum field theories have a natural interpretation as low energy approximations of string theory. The matter content of low-energy string theory is fixed provided a space-time background is chosen and the precise configuration of branes is specified. The matter content then derives from the lowest level excitations of the strings moving in such a background. String theory will be discussed in more detail below.

Let me pause here for a moment and briefly elaborate on the notion of a brane. Branes are objects in string theory that can have $p = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ spatial dimensions. There are two types of strings: open and closed strings. The endpoints of an open string are attached to a brane. Those branes on which an open string is ending

[1] Due to the uncertainty relation in energy, quantum theories, both relativistic and non-relativistic, know, in principle, about all energy scales. A quantum theory therefore has the important property of being able to predict its own regime of validity.

(not all branes are of this type) are called Dirichlet p-branes or Dp-branes for short. The endpoints of a string can be charged and from the point of view of a Dp-brane this charge is a point particle leading to a particular form of electrodynamics on the brane. When multiple Dp-branes are coinciding the theory on the set of coinciding branes is a particular form of a non-Abelian gauge theory. Branes have played a crucial role in many of the successes achieved by string theory such as the counting of black hole microstates and the AdS/CFT correspondence that relates a gauge theory (without gravity) to a string theory (containing gravity). Further, branes have proven to be of fundamental importance in string theory phenomenology such as 4-dimensional models of particle physics/cosmology obtained from string theory.

Let us go back to string theory as a top-down theory. Although the properties of string theory, that are listed in the paragraph preceding the last paragraph, may appear appealing from the point of view of a top-down theoretician they provide no guiding principles in attempts to construct out of string theory the physics of elementary particles and/or cosmology. Basically, one must, in order to describe low energy 4-dimensional physics, decide how to embed the 4-dimensional theory in the 11-dimensional framework and at the same time produce the right matter content of for example the standard model from a unique 11-dimensional theory while breaking supersymmetry somewhere along the way. Needless to say this is a very difficult task that is furthermore complicated by the high degree of non-uniqueness in which 4-dimensional physics can be obtained from string theory. This high degree of non-uniqueness in producing 4-dimensional physics from string theory is encoded in a concept that has been dubbed the string theory landscape. What the right approach is towards such a concept is the subject of much current debate.

Before discussing the subject matter of this thesis let me give a brief and by no means complete overview of string theory.

There exist five perturbatively defined string theories that go under the names: type I, type IIA, type IIB, heterotic $SO(32)$ and heterotic $E_8 \times E_8$ superstring theory. The type I string theory is a theory of both open and closed unoriented (meaning no world-sheet directionality exists) strings and possesses 16 space-time supersymmetries. The type IIA and IIB string theories are theories of closed oriented strings with 32 space-time supersymmetries. The closed strings of the type IIA theory have right and left moving world-sheet fermions of the same chirality while for the type IIB string they have opposite chirality, so that the type IIA world-sheet theory is chiral and the IIB world-sheet theory is non-chiral. The two heterotic superstring theories contain oriented closed strings with a left moving sector of world-sheet fields that is identical to that of the type II theories while the right movers are obtained by compactifying the 26-dimensional bosonic string theory on a 16-dimensional torus all of whose radii are equal. The 24 bosonic coordinates (world-sheet scalars) of the 26-dimensional bosonic string are divided into two groups: 8 bosonic world-sheet scalars that describe the position of the string in the 10-dimensional (uncompactified) space-time and 16 bosonic modes that take values in the 16-dimensional torus. The groups $SO(32)$ and $E_8 \times E_8$ determine the form taken by these 16 bosonic modes and their momenta. The heterotic theories both possess 16 space-time supersymmetries.

The type IIA and type IIB string theories contain besides closed strings also a spectrum of Dp-branes. As mentioned earlier open strings have the property that their endpoints must end on a Dp-brane and hence type IIA and type IIB string theory in the presence of Dp-branes become theories of both open and closed strings.

The spectrum of excitations of the above five string theories consists of fields that are organized into supermultiplets. The coupling of the string is described by a massless dynamical field, the dilaton. There exist field theories, called supergravity theories, with 16 or 32 supersymmetries in 10-dimensional space-time, whose fields are organized in the same supermultiplets as is the case for the massless excitations of the above-mentioned string theories. It turns out that string theory and supergravity are related as will now be discussed.

Each of the above-mentioned five string theories are well-defined in a perturbative setting. When the massless fields form a background satisfying the supergravity equations of motion then the dynamics of a string on such a background can be defined perturbatively in regions where the string coupling g_s is small.

The strings all have a finite length that is inversely related to the tension of the string. In a regime in which the string length is small compared to other distance scales in the theory it is possible to describe the string dynamics (of the massless excitations) in terms of an effective field theory. To obtain a field theory description one first expands the string theory in g_s after which one expands it further in α' (the square root of the string length). By tree level string theory is meant that the expansion in g_s is truncated right after the leading contribution (tree-level string scattering diagrams). By further truncating the α' expansion of the tree-level string diagrams keeping only the leading terms one obtains supergravity as the effective description.

This may seem as a tremendous oversimplification of the physics of string theory, and to a large extent it is, but it has turned out that certain statements derived from supergravity are protected by supersymmetry and can be turned into statements that are correct at all order in α'. Further, as mentioned above, field theory solutions to supergravity are starting points of a background on which string theory can be formulated (at least when g_s is small in the supergravity background). Even more so, this simple field theory approach can be used to gain insights into some non-perturbative aspects of the theory. Branes are a good example of this. The Dp-branes of perturbative string theory have a mass that is inversely proportional to the string coupling and are therefore extremely massive objects in the perturbative regime. These Dp-branes show up in supergravity as certain p-brane solutions, i.e. solutions describing extended objects that extend in p spatial directions [1]. Therefore, solutions to the supergravity equations of motion can be used to study non-perturbative aspects of the theory.

Consistent theories of quantum gravity must be free of gauge anomalies. A gravitational anomaly would imply that at the quantum level local Lorentz invariance, or what is the same, general coordinate invariance is broken. If the theories contain furthermore besides gravity Yang–Mills fields then again consistency requires there to be no gauge anomalies present in the theory. If the theory contains space-time chiral fermions or chiral gauge fields (gauge fields whose fields strengths are self-dual) there can be associated chiral anomalies. In the years 1983 and 1984 (the first string theory revolution) it has been shown that the supergravity approximations of the five different string theories are free of anomalies. The type IIB supergravity has chiral fermions and a chiral 4-form potential leading to chiral, gravitational and mixed anomalies. It was shown in [2] that all these anomalies cancel. The type IIA supergravity theory is non-chiral (it contains two chiral fermions of opposite chirality), does not have non-Abelian gauge fields and no self-dual gauge fields. It is therefore trivially free anomalies. The fact that the type I theory and the two heterotic theories that all contain chiral fermions and non-Abelian gauge fields are free of anomalies was shown in [3].

Approximately ten years after the first string theory revolution it was realized that the five different string theories are all related via so-called duality transformations. Further, in this web of dualities an 11-dimensional theory plays an important role. The string coupling of the type IIA supergravity theory can be interpreted as coming from the circle reduction of the unique 11-dimensional supergravity with 32 supersymmetries. Weakly coupled type IIA string theory thus corresponds to a very small circle extending in the eleventh dimension. But when the IIA string coupling becomes large the circle decompactifies and an 11-dimensional theory appears [4, 5]. The type IIA superstring becomes the 11-dimensional supermembrane, that was first introduced in [6]. Without going into the details some of the most common string-string dualities are listed. Type IIA string theory on a 9-dimensional space-time times a circle of radius R is said to be T-dual to the type IIB string theory on a 9-dimensional space-time times a circle of radius α'/R [7,8]. The type IIB string theory in the strong coupling limit is dual to itself, a duality referred to as S-duality which will play an important role in this thesis. The type IIB theory can be related to the type I theory through the introduction of 32 D9-branes and one so-called orientifold O9-plane (see section 2.5). The strong coupling limit of the type I theory is dual to weakly coupled heterotic $SO(32)$. Finally M-theory on the orbifold $S^1/\mathbb{Z}_2$ which is an interval with a hyperplane at each end gives heterotic $E_8 \times E_8$ [9, 10] and the list goes on. Strongly coupled string theory is a subject that has been given the name of M-theory whose low energy approximation must be the 11-dimensional supergravity theory since this latter theory is unique.

To summarize string theory provides a perturbatively well-defined description of quantum gravity. The five different string theories are free of anomalies. At the non-perturbative level all the five string theories are related to a unique theory, called M-theory, whose low energy approximation is eleven-dimensional supergravity. Further, besides being a quantum theory of gravity string theory also naturally contains Yang–Mills sectors. Hence, the full non-perturbative version is expected to provide a unique theory of quantum gravity, in which the dimensionality of space-time is predicted by the theory itself and which naturally contains Yang–Mills gauge interactions. At the perturbative level the theory contains only one dimensionful parameter, the string length, but this may be an artifact of the perturbative description. The input on which string theory is founded is special relativity and quantum theory. The dynamical objects of the theory are strings and branes.

Besides having an understanding of the laws that govern the dynamics of the fundamental objects of the theory it is equally important to understand the structure of the vacua of the theory. In recent years it has become gradually more and more apparent that the vacuum structure of string theory/M-theory is overwhelmingly vast and complex. The vacua of string theory/M-theory are collectively referred to as the landscape of the theory [11, 12]. The notion and even the definition of the landscape is at present not well-formulated. But, it is an unsurmountable notion that needs to be understood before one can truly hope to test the theory against experiment.

A theory that in part addresses both the vacuum structure of M-theory and parts of the M-theory moduli space is F-theory that was first introduced in [13]. It was mentioned that the type IIB theory in the strong coupling limit is dual to itself, a property known as S-duality. The type IIB theory actually possesses a discrete group of duality transformations, the group $SL(2,\mathbb{Z})$ [14]. The coupling constant of the type IIB theory is not a real massless field as is the case in the other four string theories, but rather a massless complex field, denoted by τ. Besides being a coupling constant τ is at the same time a field that couples to branes, viz. 7-branes and instantons. The 7-branes are described by F-theory and the instantons can be thought of as the electric/magnetic dual partners of the 7-branes. What F-theory is will be explained in section 2.6. Compactifications of F-theory down to four dimensions provide interesting insights into the 4-dimensional vacuum structure of the IIB theory (see for example [15]). The landscape of F-theory vacua will not be discussed in this thesis.

F-theory provides furthermore a means to study the IIB theory in the non-perturbative regime where the complex coupling τ is of order unity. This should be contrasted with the perturbative regime where τ is such that the string coupling g_s is small. F-theory will be used in this thesis to argue for the existence of novel types of branes, called Q7-branes. The notion of a Q7-brane leads to the notion of Q-instantons that are related to Q7-branes by electro-magnetic duality. Understanding the world-volume theory of the Q7-branes and the role of the Q-instantons provides a means to study the IIB theory in, so far, poorly investigated corners of its moduli space. By moduli space is meant the set of inequivalent values of the complex type IIB coupling constant τ. This work aims at improving the understanding of the complete set of so-called one-half BPS branes (preserving 16 supersymmetries) of the type IIB supergravity theory.

From the set of one-half BPS objects that are present in type IIB supergravity almost all the branes have been accounted for in type IIB string theory. These are the branes that are referred as the (p', q') p-branes with p' and q' relatively prime integers. These are p-dimensional branes on which a (p', q') string is ending. The above-mentioned Q7-branes and Q-instantons are not of this type.

This thesis is organized as follows. Chapter 1 gives an overview of type IIB supergravity. In chapter 2 the branes of the IIB theory are reviewed and the notion of F-theory is introduced. Chapter 3 deals with the subject of Q7-branes and their possible F-theory interpretation. The Q-instantons are discussed in chapter 4. The chapter on 7-branes is based on [16,17] and the chapter on instantons is based on [18].